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Multifractal analysis in hydrology

Application to time series

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Introduction

The aim of this technical note is to provide a simplified description of the procedures to be used to conduct an analysis of hydrological data using multifractal theory. It was produced as part of a post-doctoral study, codirected by hydrologists from Cemagref and fractal mathematicians from Cereve (see Ackowledgements). The links between multifractal theory and the hydrological statistical framework are underlined. Readers with a knowledge of hydrology will find in it the normal problems encountered in this field, related to the large number of possible observation scales, the great variability in phenomena down to the smallest scales, the occurrence of rare extreme events with a much higher severity compared to the system's "normal" state, the non-linearity, or even the multiplicative effects of relations between the variables of the hydrological system. This often leads to the choice of a simplified description of the system, rather than one based on physical models. The reader will find a theory and its statistical tools that deal with these problems and that attempt to model them in compliance with fundamental physical properties related to large-scale interactions. The article will sometimes bypass complex mathematical developments, but some competence in advanced statistics is however desirable. Readers wanting to know more can refer to the reference list to obtain further understanding in this field.

We will briefly summarise multifractal theory and its main properties. We will describe the procedures for identifying scale invariance properties, the core concept of this theory and on this basis we will give a first definition of multifractal properties in statistical terms. The general properties of multifractal series or fields and the behaviours of extremes analysed within the context of this theory will be described. We will then study a type of process that confirms, by a stochastic construction, the stated properties: multiplicative cascades. We will only deal with one of the models among those which can be found in the literature: the universal model. This model summarises the variability observed at all scales by an analytical formula and a reduced number of parameters to be fitted to the data being processed. Finally, the Chapter 2 is devoted to examples of applications to rainfall and discharge time series.

Throughout the article, aspects of the application of multifractal theory to hydrology which are still being studied will be mentioned, hoping to demonstrate the scientific interest of these tools.

Chapter 1
Multifractal theory

Multifractal analysis is an appropriate tool for processing and modelling data series showing a strong spatio-temporal variability, especially in terms of the non-uniform characteristics of the phenomena and their extreme behaviour. It provides a synthetic modelling of the variability of the processes being analysed. In particular, it introduces the concept of scale invariance, i.e. the relation between a measurement and the scale of this measurement.

The idea of applying these concepts to the description of geophysical variables therefore naturally came to the minds of scientists. It was applied to various related fields such as geology, meteorology, biology, geomorphology, topography, radar imagery and of course hydrology. As emphasized by Chow (1988), hydrologists must often describe systems on which various processes act at different time scales (from seconds to centuries) and different spatial scales (from millimeters to the global scale).

How can phenomena be observed at different scales?

The scale ratio

As a first stage we will restrict ourselves to general definitions concerning the scales or resolutions at which the phenomena are measured or observed. Questions of a general nature related to the scale of analysis of temporally and/or spatially dynamic systems frequently arise. At what scale should the system be observed? How do the system's characteristics change if it is observed at different scales? Can information measured at one scale be transferred to another scale? If so, how?

We assume, that the process ε we are interested in can be observed at different scales of observation, these scales being related by the scale ratio λ defined as follows:

Definition 1. $\lambda = \dfrac{T_1}{T_2}$,

where T_2 represents the larger γscale of study, and T_1 a finer scale at which the phenomenon is observed. This definition is entirely general; the value of the ratio λ quantifies the ratio between two scales of measurement.

Observations on and scales of hydrological processes

The problem of scales is particularly important in the field of hydrology, where normally a series or field of measurements is available (e.g. rainfall, soil moisture content, discharge, etc.) that vary greatly in space and time. These series are representative of different overlapping phenomena that counteract one another in a non-linear manner and which act at different temporal and spatial scales.

For reasons of simplicity and to make the demonstration clearer we will limit ourselves in this technical note to a one-dimensional process $\varepsilon(t)$ which varies along a time axis t. The target time series in hydrology will be mainly rainfall time series at a meteorological station and discharge measurements recorded at a river section, which are both renowned for their great temporal variability. The extension of the concepts to a multidimensional space could in principle be done without any supplementary difficulties.

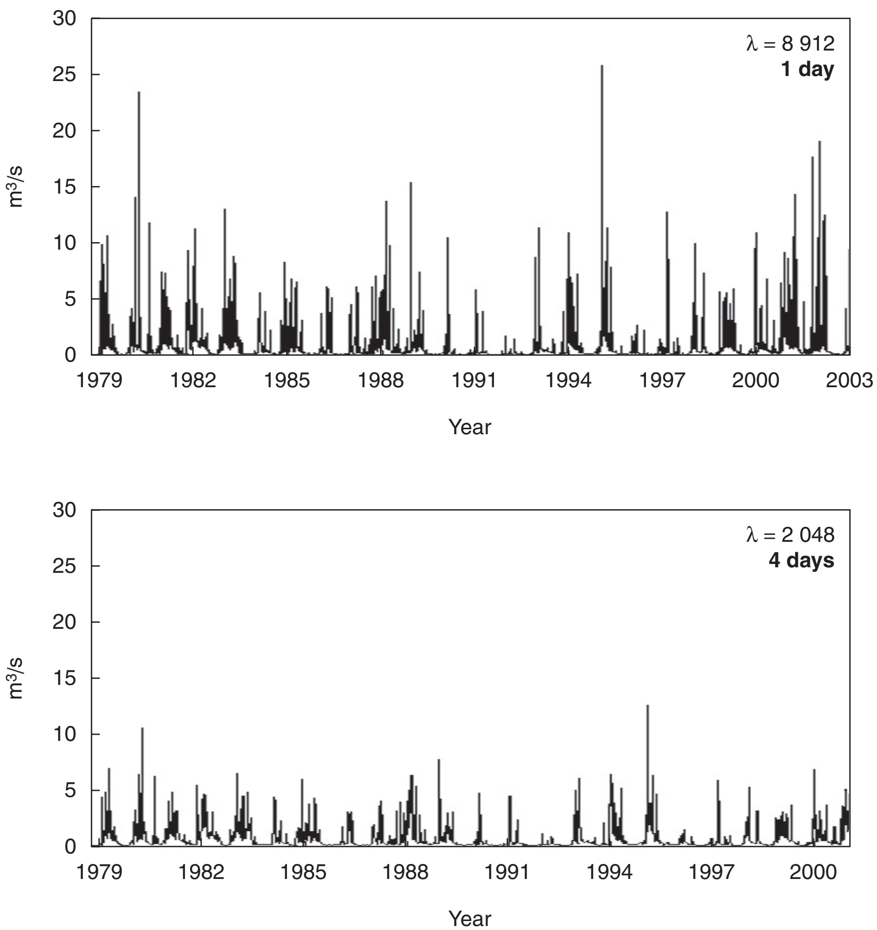

Figure 1.1. Variability of mean discharge measurements at different time scales, from 1 day (top figure) to 256 days (lower) at the Le Theil gauging station on the river Orgeval.